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See also the Clay Mathematics Institute website:

- The Poincaré conjecture and Dr. Perelman's work: http://www.claymath.org/poincare
- The Millennium Prizes: http://www.claymath.org/millennium/
- Full text: http://www.claymath.org/poincare/millenniumprize.pdf

First Clay Mathematics Institute Millennium Prize Announced Today

Prize for Resolution of the Poincaré Conjecture a Awarded to Dr. Grigoriy Perelman

The Clay Mathematics Institute (CMI) announces today that Dr. Grigoriy Perelman of St. Petersburg, Russia, is the recipient of the Millennium Prize for resolution of the Poincaré conjecture. The citation for the award reads:

> *The Clay Mathematics Institute hereby awards the Millennium Prize for resolution of the Poincaré conjecture to Grigoriy Perelman.*

The Poincaré conjecture is one of the seven Millennium Prize Problems established by CMI in 2000. The Prizes were conceived to record some of the most difficult problems with which mathematicians were grappling at the turn of the second millennium; to elevate in the consciousness of the general public the fact that in mathematics, the frontier is still open and abounds in important unsolved problems; to emphasize the importance of working towards a solution of the deepest, most difficult problems; and to recognize achievement in mathematics of historical magnitude.

The award of the Millennium Prize to Dr. Perelman was made in accord with their governing rules: recommendation first by a Special Advisory Committee (Simon Donaldson, David Gabai, Mikhail Gromov, Terence Tao, and Andrew Wiles), then by the CMI Scientific Advisory Board (James Carlson, Simon Donaldson, Gregory Margulis, Richard Melrose, Yum-Tong Siu, and Andrew Wiles), with final decision by the Board of Directors (Landon T. Clay, Lavinia D. Clay, and Thomas M. Clay).

James Carlson, President of CMI, said today that "resolution of the Poincaré conjecture by Grigoriy Perelman brings to a close the century-long quest for the solution. It is a major advance in the history of mathematics that will long be remembered." Carlson went on to announce that CMI and the Institut Henri Poincaré (IHP) will hold a conference to celebrate the Poincaré conjecture and its resolution June 8 and 9 in Paris. The program will be posted on www.claymath.org. In addition, on June 7, there will be a press briefing and public lecture by Etienne Ghys at the Institut Océanographique, near the IHP.

Reached at his office at Imperial College, London for his reaction, Fields Medalist Dr. Simon Donaldson said, "I feel that Poincaré would have been very satisfied to know both about the profound influence his conjecture has had on the development of topology over the last century and the surprising way in which the problem was solved, making essential use of partial differential equations and differential geometry.

Poincaré's conjecture and Perelman's proof

Formulated in 1904 by the French mathematician Henri Poincaré, the conjecture is fundamental to achieving an understanding of three-dimensional shapes (compact manifolds). The simplest of these shapes is the three-dimensional sphere. It is contained in four-dimensional space, and is defined as the set of points at a fixed distance from a given point, just as the two-dimensional sphere (skin of an orange or surface of the earth) is defined as the set of points in three-dimensional space at a fixed distance from a given point (the center).

Since we cannot directly visualize objects in *n*-dimensional space, Poincaré asked whether there is a test for recognizing when a shape is the three-sphere by performing measurements and other operations *inside* the shape. The goal was to recognize all three-spheres even though they may be highly distorted. Poincaré found the right test (simple connectivity, see below). However, no one before Perelman was able to show that the test *guaranteed* that the given shape was in fact a three-sphere.

In the last century, there were many attempts to prove, and also to disprove, the Poincaré conjecture using the methods of topology. Around 1982, however, a new line of attack was opened. This was the Ricci flow method pioneered and developed by Richard Hamilton. It was based on a differential equation related to the one introduced by Joseph Fourier 160 years earlier to study the conduction of heat. With the Ricci flow equation, Hamilton obtained a series of spectacular results in geometry. However, progress in applying it to the conjecture eventually came to a standstill, largely because formation of singularities, akin to formation of black holes in the evolution of the cosmos, defied mathematical understanding.

Perelman's breakthrough proof of the Poincaré conjecture was made possible by a number of new elements. Perelman achieved a complete understanding of singularity formation in Ricci flow as well as the way parts of the shape collapse onto lower-dimensional spaces. He introduced a new quantity, the entropy, which decreases as time increases during Ricci flow, signaling an increase in the degree of geometric order in the underlying shape. He introduced a related local quantity, the L-functional, and he used the theories of Cheeger and Aleksandrov to understand limits of spaces changing under Ricci flow. He was also able to show that the time between formation of singularities could not become smaller and smaller, with singularities becoming spaced so closely – infinitesimally close – that the Ricci flow method would no longer apply. Perelman deployed his new ideas and methods with great technical mastery and described the results he obtained with elegant brevity. Mathematics has been deeply enriched.

Some other reactions

Fields medalist Stephen Smale, who solved the analogue of the Poincaré conjecture for spheres of dimension five or more, commented that: "Fifty years ago I was working on Poincaré's conjecture and thus hold a long-standing appreciation for this beautiful and difficult problem. The final solution by Grigoriy Perelman is a great event in the history of mathematics."

Donal O'Shea, Professor of Mathematics at Mt. Holyoke College and author of *The Poincaré Conjecture,* noted: "Poincaré altered twentieth-century mathematics by teaching us how to think about the idealized shapes that model our cosmos. It is very satisfying and deeply inspiring that Perelman's unexpected solution to the Poincaré conjecture, arguably the most basic question about such shapes, offers to do the same for the coming century.