The Life and Work of Famous Chinese Mathematician Loo-keng Hua

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Professor L. K. Hua had a dynamic career. He is a distinguished and outstanding mathematician in the world as well as in China. He is the founder of the modern mathematics in china, and led the Chinese mathematics for a long time. His life is eventful and interesting from a self-taught individual into a world-reknown mathematician. His story educated and encouraged several generations in China. He is so famous in China that if you talk about mathematics or mathematicians of China to some people in the world, usually they will speak about L. K. Hua. If you have an opportunity to visit the campus of schools or universities in China, you may find the picture of Professor Hua in their classroom, and quoted some of what he said with his picture, for example: "Cleverness comes from study, natural talent lies in persistence", "Fist never need to be thrown, tune never need to be hummed", etc. If you go to a post office to buy a post stamp, perhaps you get a stamp with the picture of Hua. If you walk around on the campus of some institutions in China, you may find his statues. For example, at the campus of Tsing-Hua University, at the campus of University of Science and Technology of China, at the Institute of Mathematics, Academia Sinica, at his hometown, Jintan county, etc. And you may find also some schools that have been named by his name, and some Chinese mathematics competitions have been named by his name, etc. If you watch TV at your hotel in China, you may find some TV shows which describe his life or his story. However, Professor Hua had big influence in China.

Now I like to give a brief outline of his life.

Professor L. K. Hua was born on November 12, 1910 in Jintan county in the south of Jiangsu Province of China. His father managed a small family grocery store. How small this family grocery store was? Hua told me that most customers were poor people. They always bought only one cigarette and had to borrow a light to light it with. Hua's family was too poor to allow him to enrol in senior middle school when he graduated from Jintan junior middle school. He then attended the Shanghai Chung-Hua Vocational School where he completed one and a

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half years of its two-year accounting course. He was forced by the poor circumstances of his family to leave school at the age of fifteen and return home to help his father in the family shop. He could only learn mathematics from few books in his spare time. He was so interested in mathematics that he could not pay full attention to the shop. Of course, his father was unhappy with him and often threatened to burn his books.

In 1927, Hua got a job as a clerk in Jintan middle school and married Wu Xiao-yuan. They had a daughter in the following year, and there are also three sons and two more daughters. In 1928, Hua was struck by typhoid, followed by arthritis which burdened him for life with a lame left leg.

Hua published his first paper in the journal *Science* (Shanghai) in 1929. His second paper *On the incorrectness of Su Jia Ju's paper* appeared in the same journal in the following year. This paper was noticed by Professor C. L. Hsiung, the chairman of the Department of Mathematics of Tsing Hua University in Beijing. Of course, Hsiung had never heard of anyone called Loo-keng Hua. Later, a Jintan born teacher in this department, P. C. Tang, informed Hsiung that Hua was not even a middle school graduate, but a mere clerk in a small village. Hsiung was impressed and invited Hua to Tsing Hua University as a clerk in his department in 1931. Hua was appointed departmental assistant in the next year. Then he was promoted to the rank of a lecturer and made a research fellow of the China Cultural Foundation in 1934. During this period, his contemporaries at Tsing Hua who were to become distinguished mathematicians were S. S. Chern and P. L. Hse. Hua's initial research interest was in Waring's problem in number theory.

In 1936, N. Wiener recommended Hua to G. H. Hardy, and Hardy invited Hua to visit Cambridge, England. Hua had an opportunity to discuss with several young mathematicians in Cambridge, among whom were H. Davenport, T. Estermann, R. A. Rankin and E. C. Titchmarch. With at least fifteen papers written during his Cambridge period it is obvious that he benefited much from these mathematicians who remained his life long friends.

In July 1937, Japan invaded China, Tsing Hua University together with Peking University and Nanka University had to be evacuated to Kumming in Yunnan province where they formed the Southwest Associated University, and Hua returned from Cambridge to become a professor there from 1938 to 1945. His research works had broadened to include the geometry of matrices, automorphic functions, several complex variables and group theory.

In 1946, at the invitation of Soviet Academy of Sciences and Soviet International Cultural Association, Hua visited the Soviet Union for three months and he met I. M. Vinogradov and Yu. V. Linnik.

From 1947 to 1948, Hua was a visiting member of the Institute for Advanced Studies, Princeton. During 1948–1950, Hua was a professor at the University of Illinois at Urbana-Champaign.

When the new China was established in 1949, Hua returned to China with his wife and children in 1950 and took part in the preparatory work for the establishment of the Institute of Mathematics, Academia Simica. He was appointed Director of Institute when it opened in 1952, and he immediately took charge of the reconstruction program for the Institute which was to consist of sections in pure mathematics, applied mathematics, and also computing science. He paid special attention to the training of young mathematicians among whom were Jing-run Chen and Yuan Wang in number theory, Zhe-xian Wan in algebra, and Qi-keng Lu and Sheng Gong in several complex variables. For their benefit, and also for other Chinese mathematicians, he embarked on the writing of a series of books. All these books are translated into English and some other foreign languages.

Hua was subjected to harassing interrogation during the "Cultural Revolution" in 1966. His house was searched by the "Red Guards" and many of his manuscripts were confiscated and irretrievably lost.

In 1960, Hua was appointed as Vice President of the University of Science and Technology of China and began work on applied mathematics as well as pure mathematics, particularly on the application of number theory to numerical integration in multidimensional space. He also popularized the "Optimum seeking method" and the "Critical path method" in Chinese factories and industry departments.

The "Cultural Revolution" came to an end in 1978 and China adopted an open policy in 1979. He published two more books. His *Selected papers*, edited by H. Halberstam was published by Springer-Verlag in 1983. Hua was Vice President of the Chinese Academy of Science in 1978 and the Director of the Institute of Applied Mathematics in 1980. He was elected President of the Chinese Mathematical Society, holding this position from 1950 to 1983.

Hua received honorary doctorates from the University of Nancy (1980), the Chinese University of Hong Kong (1983) and University of Illinois (1984). He became a foreign member of U. S. Academy of Science (1983) and the Deutsche Akademie der Naturforscher Leopoldina. He was elected as a member of the Third World Academy of Sciences in 1983. He was in poor health and he continued to work in mathematics.

He died on June 12, 1985 of a heart attack at the end of his lecture at Tokyo University in Japan.

The resarch interests of Hua were broad. He gave a lot of important contributions to several branches of mathematics, especially in number theory, algebra, geometry, several complex variables and applied mathematics. Most of his results were classical and fundamental. His mathematical ideas were elegant. I think it is no enough space to say more detail about his whole mathematical work. I like to say very briefly something about his work except his contributions in several complex variables which are closely related to Clifford analysis.

Hua made a lot of very important contributions to number theory, especially, on the estimations of trigonometric sums, on Waring problems, Tarry problems, etc. He applied number theory to numerical analysis. However, one event about number theory in China should be mentioned. In 1953–1957, Hua organized a seminar program on Goldbach's problem in the Institute of Mathematics, Academia Sinica. Goldbach conjectured *(A) every even number greater than 5 is a sum of two odd primes* (we shall write this in (1,1) for simplicity), *(B) every odd number greater than 8 is a sum of three odd primes.* After Hua organized this seminar under his guidance, his students gave a lots of important contributions on this problem one after another. For example, Yuan Wang proved (3, 4) in 1956, then (2, 3) in 1957, Cheng-dong Pang proved (1, 4) in 1963, and finally, Jing-run Chen established (1, 2) in 1965, i. e., any even number is a sum of an odd prime number and a number of the product of two prime numbers. This is the best result of this kind on this problem, apart from the conjecture itself, of course.

Hua also made a lot of important contributions on algebra and geometry, especially on geometry of matrices, fields, etc.

Here we state a result of Hua on fields as an example of his contributions in algebra and geometry.

Let K be a field. A mapping $a \to a^\sigma$ of K onto itself is called a semi-automorphism if it satisfies

$$(a+b)^\sigma = a^\sigma + b^\sigma, (aba)^\sigma = a^\sigma b^\sigma a^\sigma \ \textit{and}\ 1^\sigma = 1.$$

The well-known examples of semi-automorphisms are automorplisms which satisfy $(ab)^\sigma = a^\sigma b^\sigma$, and anti-automorphisms which satisfy $(ab)^\sigma = b^\sigma a^\sigma$. An outstanding problem was whether there exists a semi-automorphism which is neither an automorphism nor an anti-automorphism. Hua settled this problem in 1949 by proving that every semi-automorphism is either an automorphism or an anti-automorphism. The fundamental theorem of projective geometry on a line over a field of characteristic $\neq 2$, namely, any one-to-one mapping carrying the

projective line over a field of characteristic $\neq 2$ onto itself and keeping harmonic relations invariant is a semi-linear transformation induced by an automorphism or an anti-automorphism, was thus derived.

Hua and Dieudonné are the founders of the geometry of matrices. They individually and independently determined most of the automorphisms of the classical groups. In general, Dieudonné adopted a method which works smoothly for n, and treated individually the cases with small n. Hua's method starts with the lowest possible n, which is usually the most difficult case, and it is easy to treat for high dimension.

Before I speak about the contributions of Hua to several complex variables, I like to quote a speech of Professor S. T. Yau in 1997. He said: *There are three most important mathematicians in China who gave the important contributions in modern matnematics which are well-known in the world. One is S. S. Chern on characteristic classes, one is L. K. Hua on several complex variables, one is Kan Feng on finite elements.* Then he mentioned why the contributions of several complex variables of Hua are the most important ones. He said, *the contributions of Hua on number theory are important, of course, but they could not influence the development of the number theory in the world. The method which he used came from abroad. But the work of Hua in several complex variables led in the world by at least 10 years.*

The first two papers of Hua in several complex variables were published in American J. of Math. in 1944 with the title *On the theory of autormorphic functions of a matrix variables I-Geometric basis, II-The classification of hypercircles under the sympletic group.* He wrote these two papers in a very bad situation. A note in these papers says *Because of the poor mail service between the U. S. and China, a number of minor changes in this paper have been made here, with the consent of the editors, by Prof. Hua's friend Dr. Hsio-fu Tian and Prof. C. L. Siegel.* Hua remarked further: *The author is great indebted to Prof. H. Weyl for sending him a copy of Siegel's paper on sympletic geometry. The author would like also to express his thanks to Prof. P. C. Tang and Prof. S. S. Chern for each sent to him one of the following two important references* (G. Giraud's paper, E. Cartan's paper). From these two notices, we find that even in the very bad situation, nearly isolated from the community of mathematicians, Hua still created very important research works. At the same time, C. L. Siegel published his well-known paper with the title *Symplectic geometry* (Amer. J. of Math. 65 (1943) 1–85). These three papers established the foundation of sympletic geometry. Siegel and Hua are the founders of this branch of mathematics. Half of a century passed until now, and sympletic geometry is still one of the very active fields in modern mathematics.

It has great influence in physics as well as in mathematics. For example, the important Hamilton system in physics has sympletic structure. It is related with dynamic systems, the three body problem and computer sciences.

The third paper of Hua in several complex variables was *On theory of Fuchsian functions of several complex variables* which was published in Annals of Math. 47 (1946). In this paper, he considered the automorphic functions on bounded homogeneous domains. No doubt, these three papers of Hua were the fundamental and classical papers on automorphic functions in several complex variables. The fourth paper of Hua in several complex variables was *On the extended space of several complex variables (I), The space of complex sphere* (Quarterly J. Math. 17 (1946)). In this paper he proved that the complex sphere (Lie sphere) is the extended space of classical domain R_{IV}.

After he went back to China in 1950, he wrote a series of papers on harmonic analysis of functions of several complex variables in the classical domains. Then he collected these results in a book with title *Harmonic analysis of functions of several complex variables in the classical domains.* The Chinese edition of this book was published in 1958. Its Russian edition was published in 1959, its English edition was published in 1963 by AMS.

In 1935, É. Cartan proved that there are six types of irreducible homogeneous bounded symmetric domains. Two of these are exceptional in that they occur only in dimension 16 and 27. The other four are the so-called "classical domains", which are defined as follows:

$$
\begin{aligned}
&R_I = \{m \times n \text{ matrices } Z \text{ satisfying } I - Z\overline{Z}' > 0\}\\
&R_{II} = \{\text{symmetric matrices } Z \text{ of order } n \text{ satisfying } I_n - Z\overline{Z}' > 0\}\\
&R_{III} = \{\text{skew symmetric matrices } Z \text{ of order } n \text{ satisfying } I_n - Z\overline{Z}' > 0\}\\
&R_{IV} = \{z = (z_1, \cdots, z_n) \in \mathbb{C}^n : |zz'|^2 + 1 - 2\bar{z}z' > 0, |\bar{z}z'| < 1\}
\end{aligned}
$$

where $'$ means transpose of a matrix.

In some sense, classical domains may be regarded as the higher dimensional analogues of the unit disk in the complex plane. Roughly speaking, in one complex variable, if you want to check whether your result is correct or not, you usually consider the result on the unit disk case. In several complex variables case, you usually consider the result in the classical domains case. In 1953, using group representation theory, Hua obtained the orthonormal system for each of the four classical domains. He gave the Bergman kernel function, Cauchy kernel and Poisson kernel of these four classical domains. Just like W. Rudin pointed out that before Hua's work, nobody knew the Cauchy kernel even if the domain is the simplest case, the unit ball. Hua's work is

fundamental and elegant. Just like Yau said, it led by at least 10 years in the world. After Hua's work, there were lots of important developments. For example, Pjatetski-Shapiro disproved the É. Cartans' conjecture: All transitive domains are symmetric. Hua and Q. K. Lu established the theory of harmonic functions for classical domains in 1958. Since the classical groups may be regarded as the characteristic manifold of classical domains, Hua, S. Gong and X. A. Zhang established the harmonic analysis on classical groups, compact Lie group and compact homogeneous spaces. S. Gong and J. K. Sun established the theory of singular integral and singular integral equations on the classical domacis. Q. K. Lu studied the heat kernel on classical domains. Xiangyu Zhou solved an important conjecture of Bogolubov and Vladiminou. S. Gong, Q. H. Yu, X. A. Zhang, S. K. Wang and T. S. Liu established the geometric function theory in several complex variables, etc.

I think I have no space to say more about the works of Hua, including his important contributions in applied mathematics.

Thank you!